

A divisibility problem for lower forms

Consider the sequence:

$7, 77, 777, 7777, 77777, \dots$

Are there any numbers in this sequence that is divisible by 693, if so, find the smallest of such numbers.



First, note that $693 = 7 \times 9 \times 11$.

All numbers in the given sequence can be divisible by 7.

In order that a number is divisible by 11, the sum of the odd digits of the number is equal to the sum of the even digits, and hence the number must be even.

In order that a number is divisible by 9, the sum of the digits must be divisible by 9. Hence the number must be of length that is the multiple of 9, since shortest sum of sevens: $7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 = 63$ is divisible by 9.

Now, the L.C.M of 2 and 9 is 18.

Hence the smallest of the number in the given sequence that is divisible by 693 is 777777777777777777 .

Yue Kwok Choy
30 May, 2015.